Chapter 6

Measuring Internal Product Attributes: Structural Complexity

Problem Statement

- How complex is the following program?

```
1: read x,y,z;
2: type = "scalene";
3: if (x == y or x == z or y == z) type - 'isosceles";
4: if (x == y and x == z) type ="equilateral";
5: if (x >= y+z or y >= x+z or z >= x+y) type ="not a triangle";
6: if (x <= 0 or y <= 0 or z <= 0) type ="bad inputs";
7: print type;
```

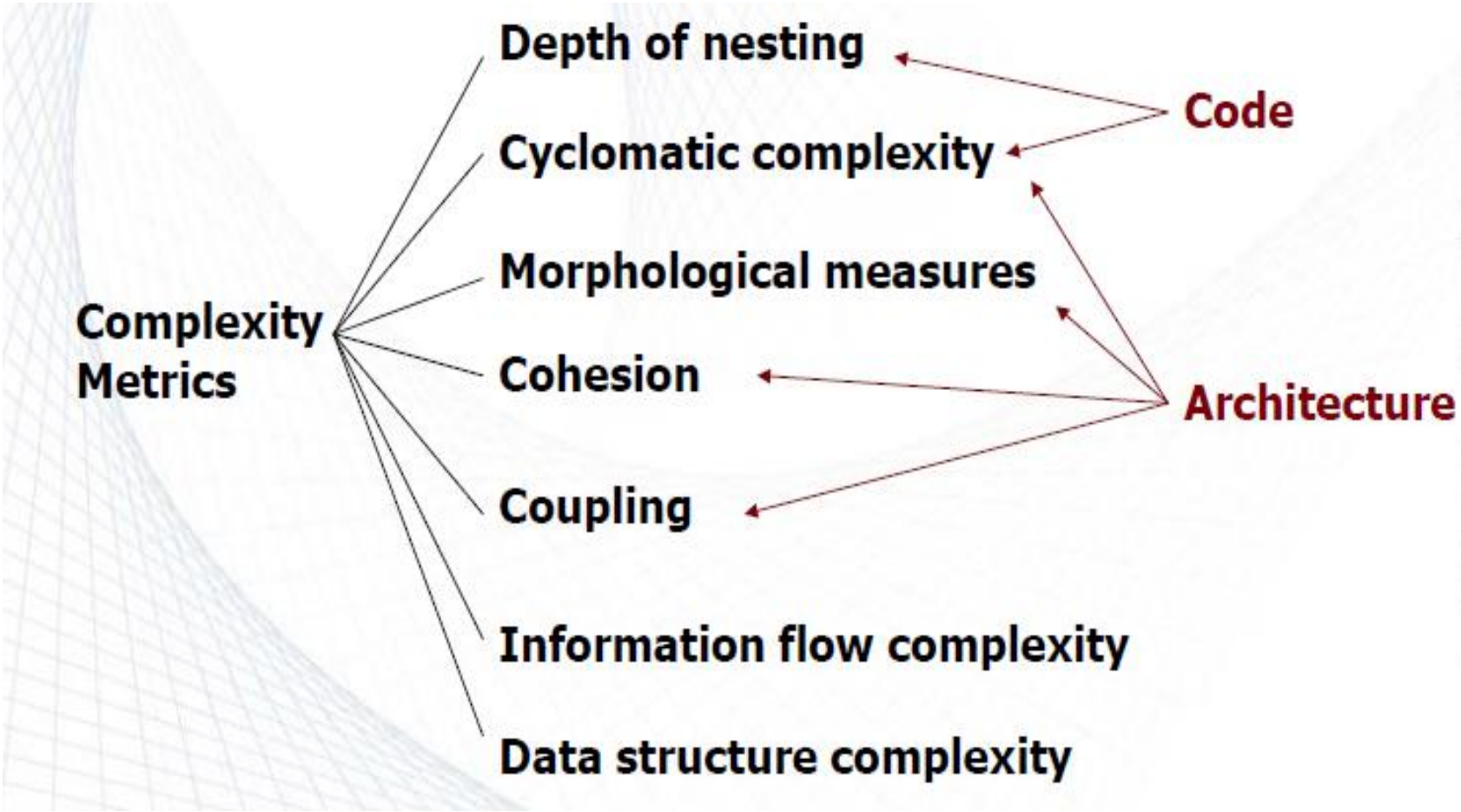
  - Is there a way to measure it?

It has something to do with program structure (branches, nesting) & flow of data

Contents

- Software structural measurement
- Control-flow structure
- Structural complexity: cyclomatic complexity
- Data flow and data structure attributes
- Architectural measurement

Software Complexity Metrics



How to Represent Program Structure?

Software structure can have 3 attributes:

- **Control-flow structure:** Sequence of execution of instructions of the program.
- **Data flow:** Keeping track of data as it is created or handled by the program.
- **Data structure:** The organization of data itself independent of the program.

Goal & Questions ...

- Q1: How to represent “structure” of a program?
- A1: Control-flow diagram
- Q2: How to define “complexity” in terms of the structure?
- A2: Cyclomatic complexity; depth of nesting

Basic Control Structure /1

- **Basic Control Structures (BCSs)** are set of essential control-flow mechanisms used for building the logical structure of the program.
- BCS types:
- Sequence: e.g., a list of instructions with no other BCSs involved.
- Selection: e.g., *if... then ... else.*
- Iteration: e.g., *do ... while* ; *for ... to ... do.*

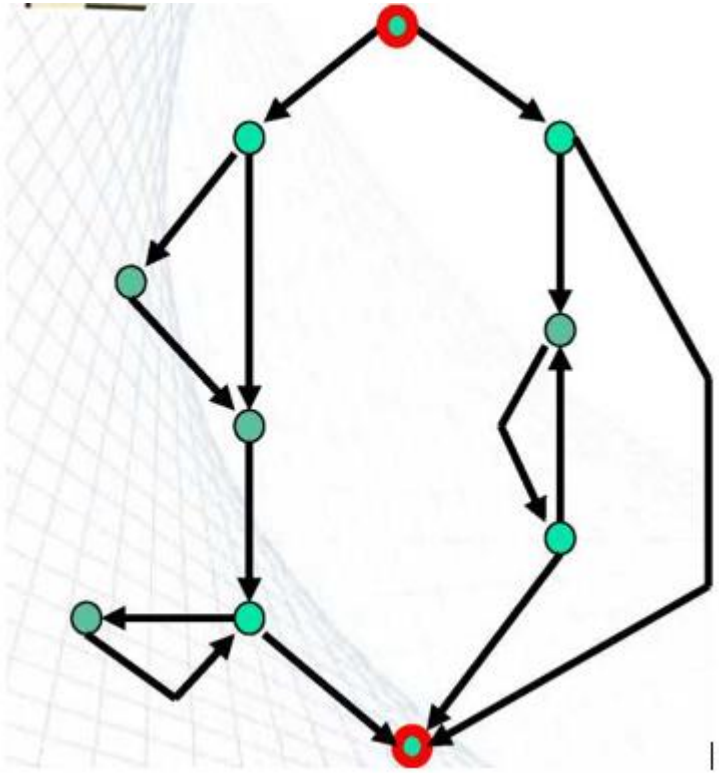
Basic Control Structure / 2

- There are other types of BCSs, (may be called advanced BCSs), such as:
  - Procedure/function/agent call
  - Recursion (self-call)
  - Interrupt
  - Concurrence

Control Flow Graph (CFG) /1

- Control flow structure is usually modeled by a directed graph (di-graph)
    - CFG = {N, A}
- Each node *n* in the set of nodes (N) corresponds to a program statement.
- Each directed arc (or directed edge) *a* in the set of arcs (A) indicates flow of control from one statement of program to another.
    - **Procedure nodes:** nodes with out-degree 1.
    - **Predicate nodes:** nodes with out-degree other than 1 and 0.
    - **Start node:** nodes with in-degree 0.
    - **Terminal (end) nodes:** nodes with out-degree 0.

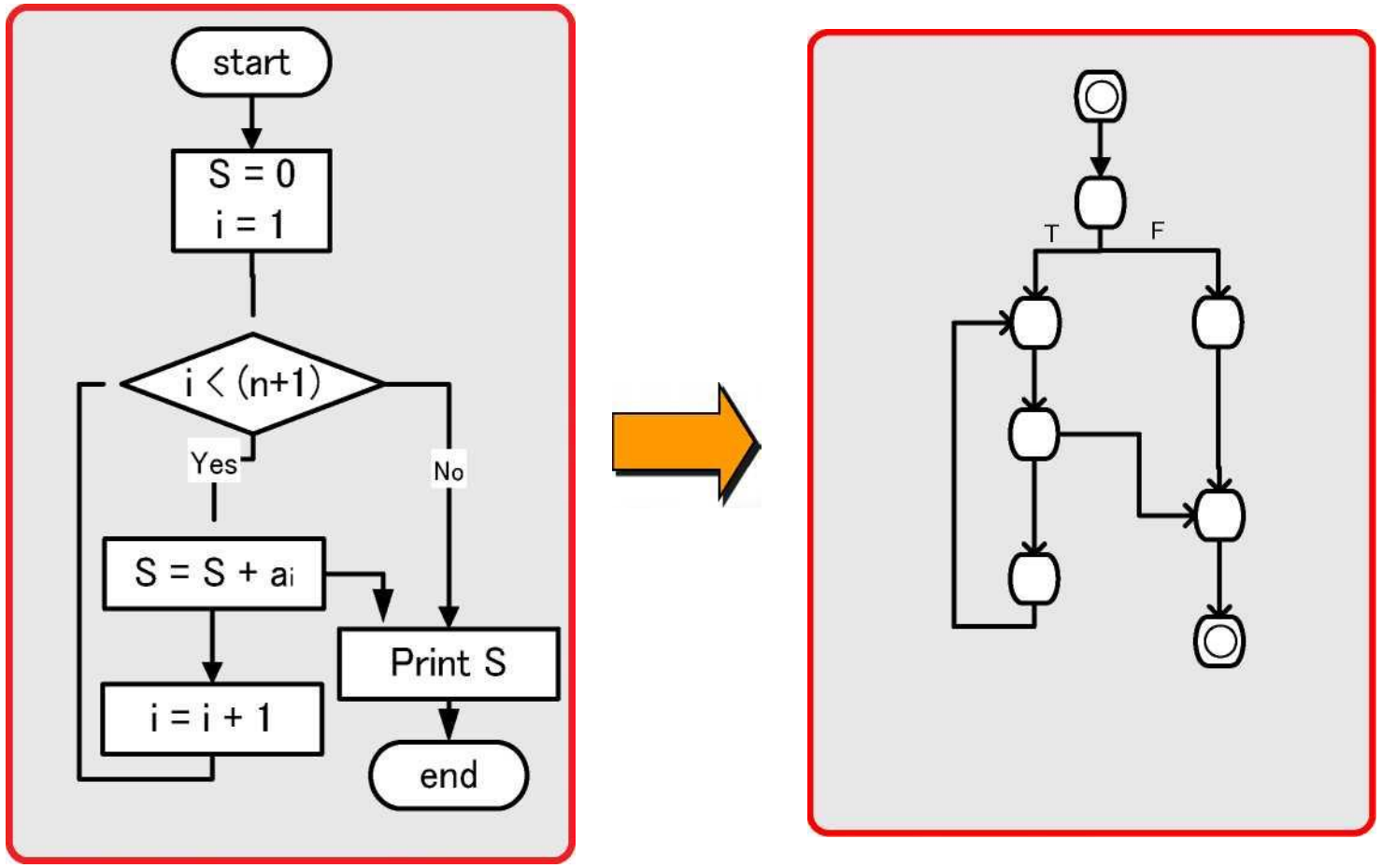
Control Flow Graph (CFG) /2



Definition [FeR97]:

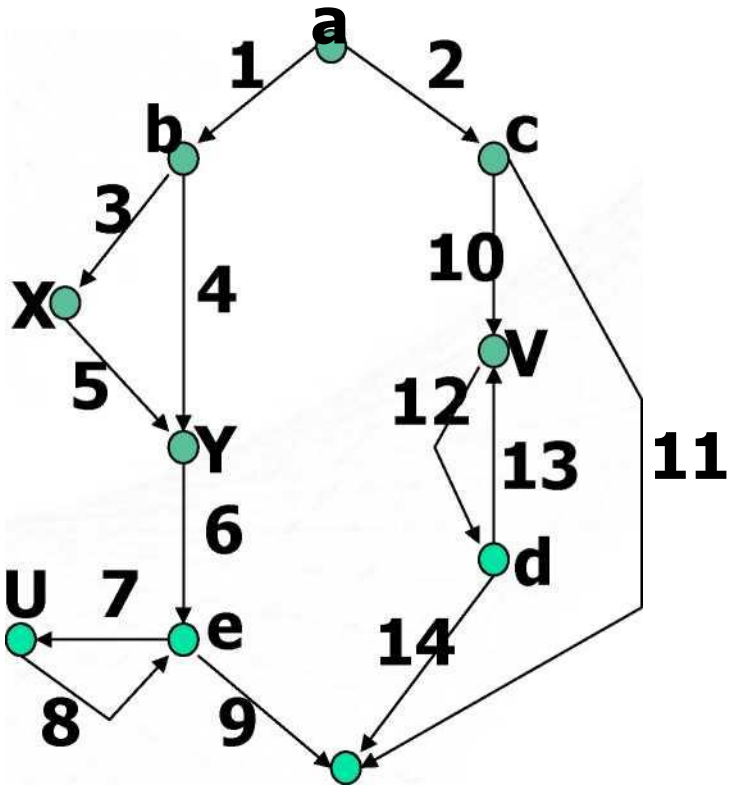
- A flowgraph is a directed graph in which two nodes, the start node and the stop node, obey special properties: the stop node has out-degree zero, and the start node has in-degree zero. Every node lies on some path from the start node to the stop node.

CFG Example 1



CFG Example 2

```
if a then if b then X
Y

    while e do U
else
    if c then
    repeat V until d
    endif
    endif
```



Control Flow Graph / 2

- The control flow graph CFG = {N, A} model for a program does not explicitly indicate how the control is transferred. The **finite-state machine (FSM)** model for CFG does.

$M = \{N, \Sigma, \delta, n_0, F\}$

N set of nodes; Σ set of input symbols (arcs)

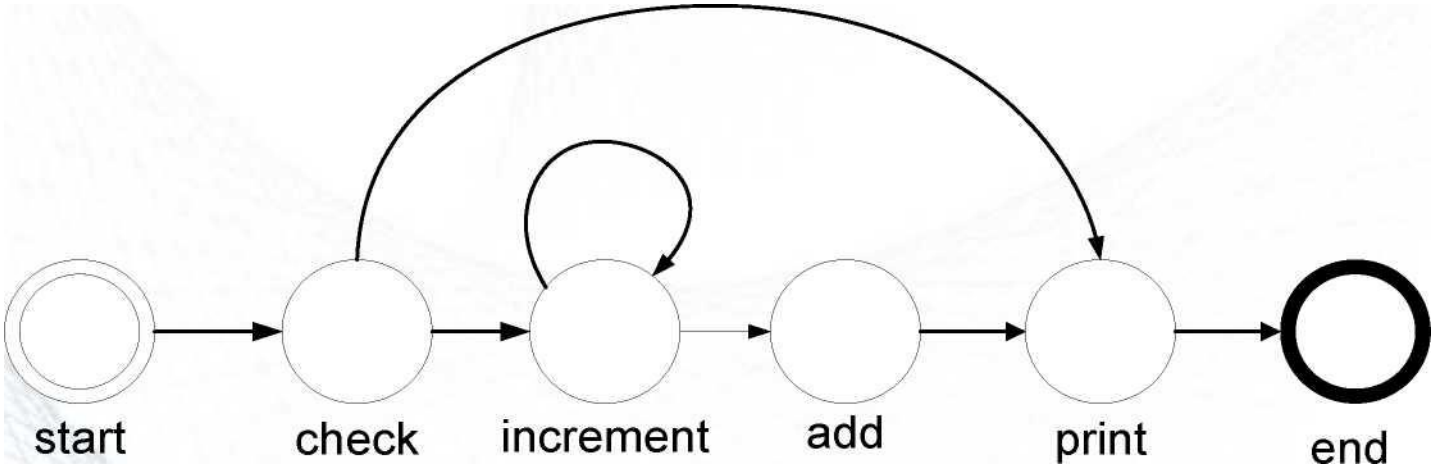
δ transition function

$\delta(p, a) = q$; $p, q \in N$ and $a \in \Sigma$

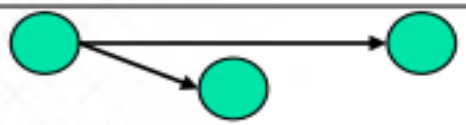
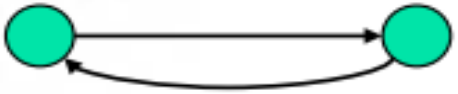
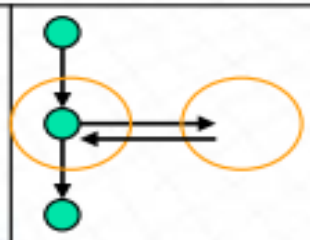
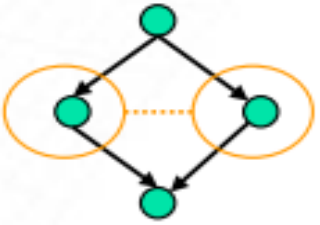
$n_0 \in$ N starting node and $F \subseteq N$ set of terminal node(s)

Example: FSM Model

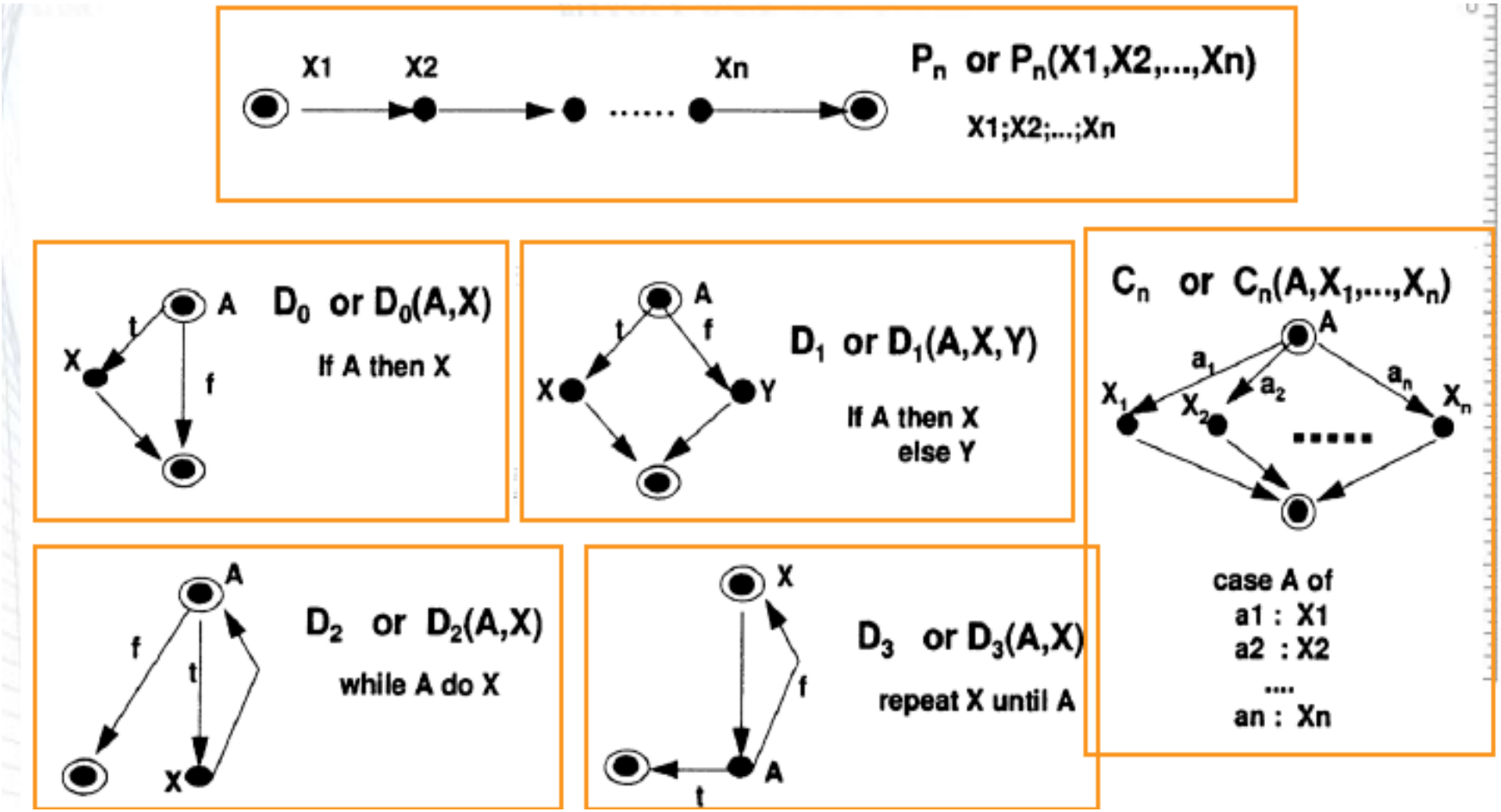
- Finite state machine model for the increment and add example



Control Flow Graph /3

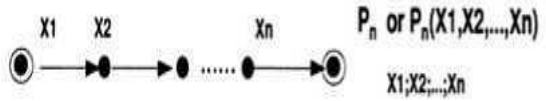
Sequence	
Selection	
Iteration	
Procedure/ function call	
Recursion	
Interrupt 	Concurrence 

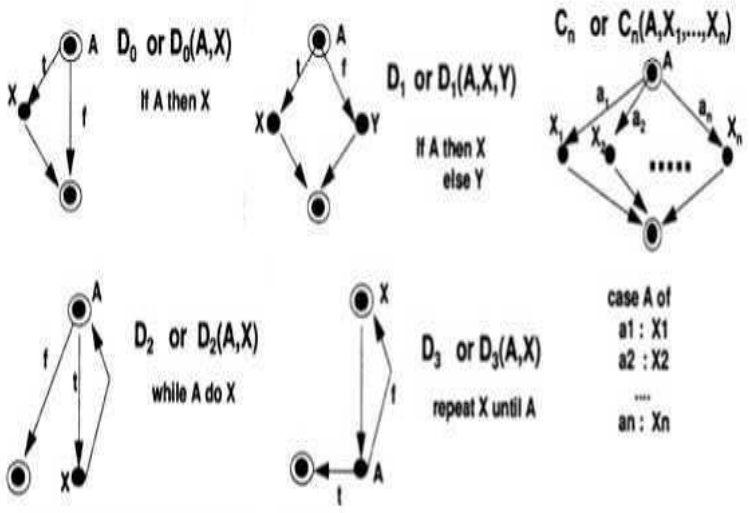
Common CFG program model



Prime Flow Graphs

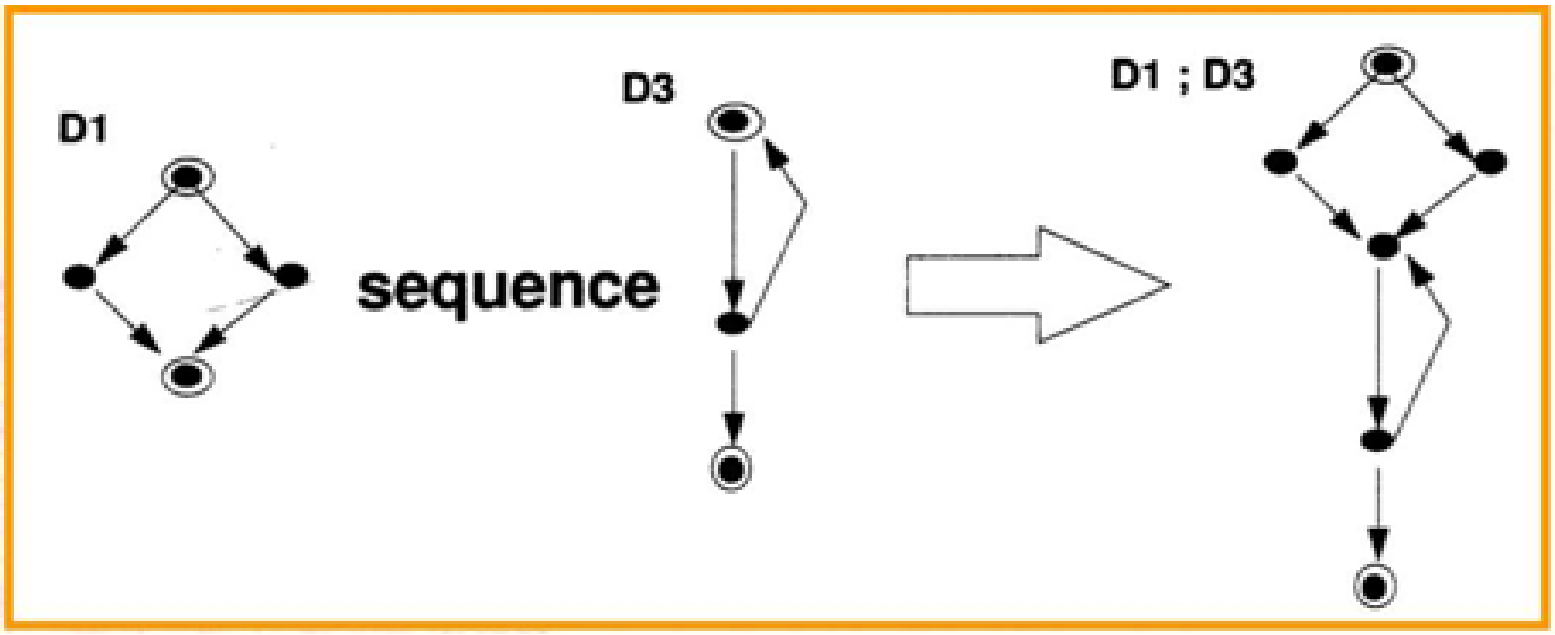
- Prime flow graphs are flow graphs that cannot be decomposed non-trivially by sequencing and nesting.
- **Examples (according to [FeP97]):**
  - Pn (sequence of n statements)
  - D0 (if-condition)
  - D1 (if-then-else-branching)
  - D2 (while-loop)
  - D3 (repeat-loop)
  - Cn (case)





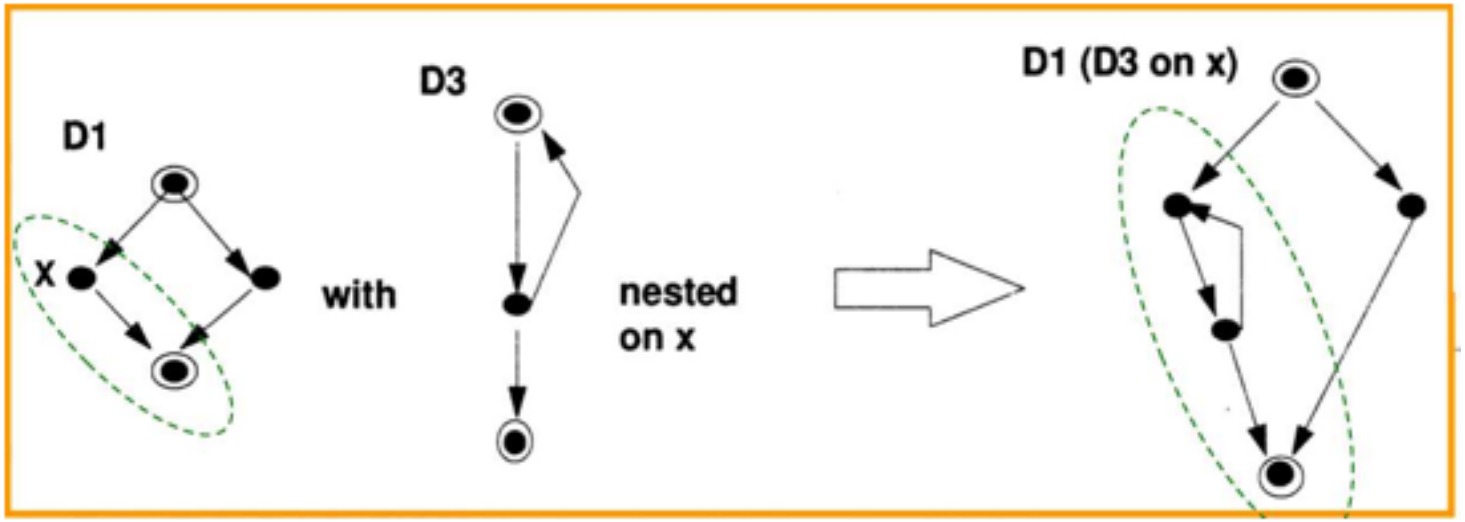
Sequencing & Nesting /1

- Let F_1 and F_2 be two flowgraphs. Then, the sequence of F_1 and F_2, (shown by F_1; F_2) is a flowgraph formed by merging the terminal node of F_1 with the start node of F_2.



Sequencing & Nesting /2

- Let F_1 and F_2 be two flow graphs. Then, the nesting of F_2 onto F_1 at x, shown by $F_1(F_2)$ is a flow graph formed from F_1 by replacing the arc from x with the whole of F_2.



S-structured Graph

- A family S of prime flow graphs is called **S-structured graph** (or **S-graph**) if it satisfies the following recursive rules:

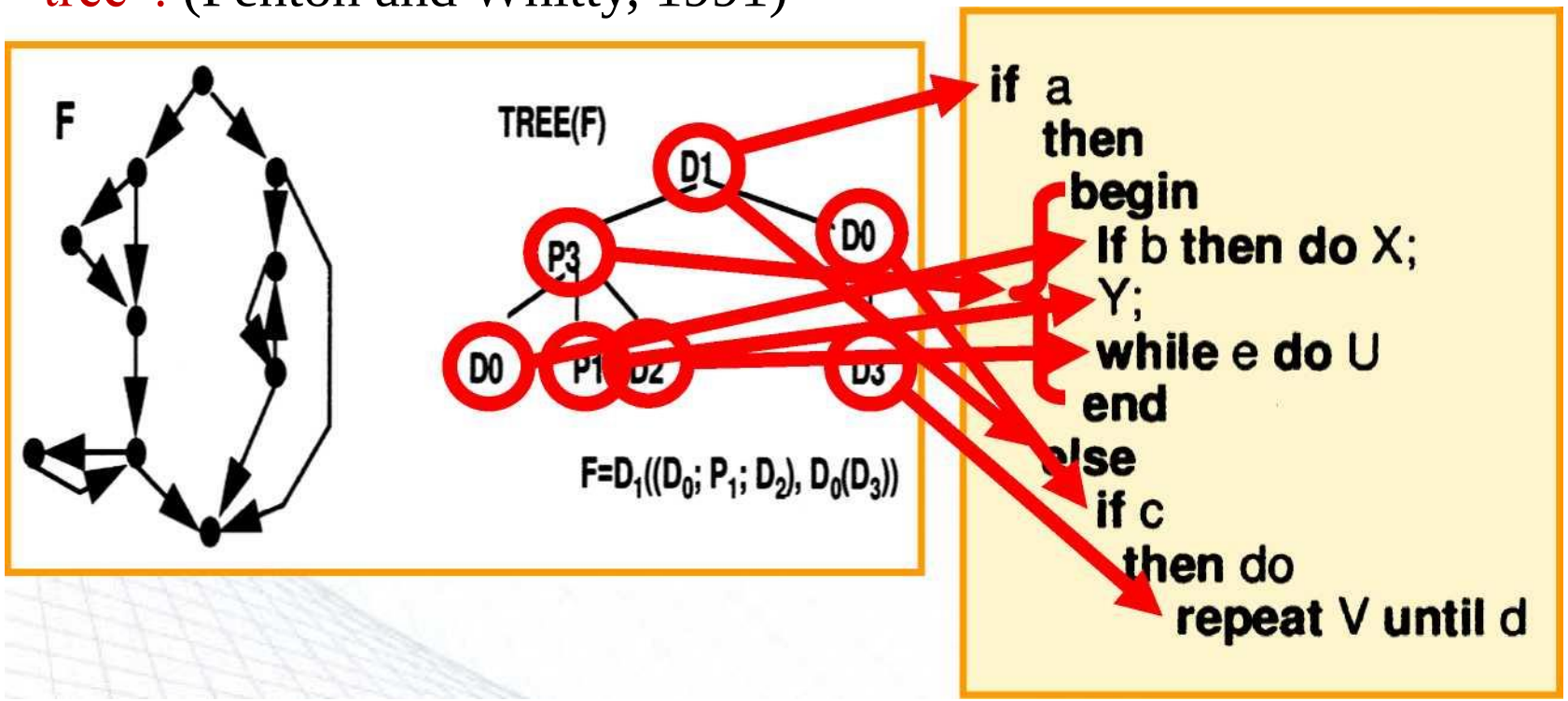
1. Each member of S is S-structured.
2. If F and G are S-structured flow graphs, so is the sequences F;G and nesting of F(G).
3. No flow graph is S-structured unless it can be generated by finite number of application of the above (step 2) rules.

S-Structured Graph - Example

- $S^D = \{P1, D0, D2,\}$
- The class of S^D-graphs is the class of flow graphs that is called D-structured (or simply: structured) in the Structured Programming literature
- Bohm and Jacopini (1966) have shown that every algorithm can be encoded as an S^D-graph (i.e., as a sequence or nesting of statements, if-conditions and while-loops)
- Although S^D is sufficient in this respect, normally if-then-else structures (D_1) and repeat-loops (D_3) are included in S^D.

Prime Decomposition

- Any flow graph can be *uniquely* decomposed into a hierarchy of sequencing and nesting primes, called "**decomposition tree**". (Fenton and Whitty, 1991)



Hierarchical Measures

- The decomposition tree is enough to measure a number of program characteristics, including:
  - Nesting factor (depth of nesting)
  - Structural complexity
  - etc.

```
if a
   then
    begin
      If b then do X;
      Y;
      while e do U
    end
   else
      if c
        then do
           repeat V until d
```

Depth of Nesting /1

- Depth of nesting *n(F)* for a flow graph *F* can be expressed in terms of:
  - **Primes:**
    - $n(P_1) = 0;\ n(P_2) = n(P_3) = \ldots = n(P_k) = 1$
    - $n(D_0) = n(D_1) = n(D_2) = n(D_3) = 1$
  - **Sequences:**
    - $n(F_1;F_2;\ldots;F_k) = max(n(F_1), n(F_2), \quad n(F_k))$
  - **Nesting:**
    - $n(F(F_1,F_2,\ldots,F_k)) = 1 + max(n(F_1), n(F_2), \ldots, n(F_k))$

Depth of Nesting /2

Example:

$F = D_1((D_0;P_1;D_2), D_0(D_3))$

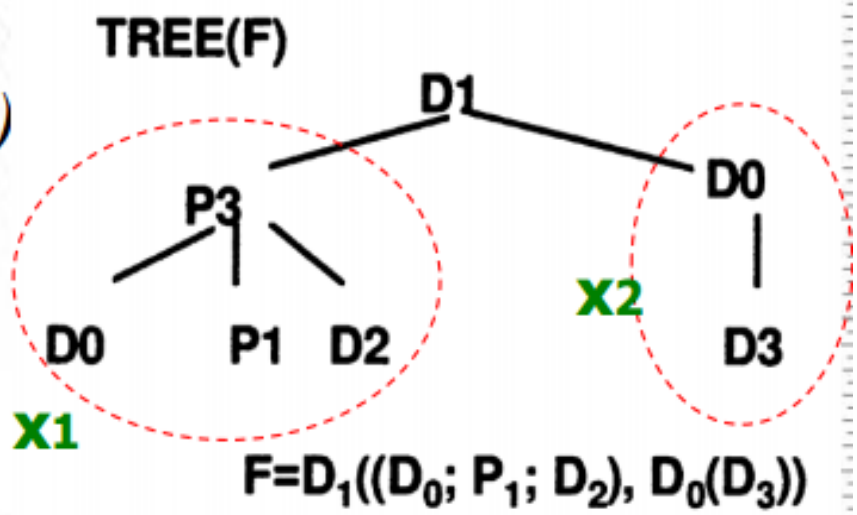
$n(F) = 1 + max(x_1, x_2)$

$x_1 = max(1,0,1)$

$x_2 = 1 + max(1) = 2$

$n(F) = 1 + max(1,2)$

$n(F) = 3$



Smaller depth of nesting indicates less complexity in coding and testing

Cyclomatic Complexity

Cyclomatic Complexity

- A program's complexity can be measured by the cyclomatic number of the program flow graph.

- The cyclomatic number can be calculated in 2 different ways:
  - Flow graph-based
  - Code-based

Cyclomatic Complexity /1

- For a program with the program flow graph *G*, the cyclomatic complexity *v(G)* is measured as:

  $$v(G) = e - n + 2p$$

  - *e* : number of edges
    - Representing branches and cycles
  - *n* : number of nodes
    - Representing block of sequential code
  - *p* : number of connected components
    - For a single component, *p=1*

Cyclomatic Complexity / 2

- For a program with the program flow graph *G*, the cyclomatic complexity *v(G)* is measured as:

$$v(G) = 1 + d$$

  - *d*: number of predicate nodes (i.e., nodes with out-degree other than 1)
    - d represents number of loops in the graph
    - or number of decision points in the program

i.e., The complexity of primes depends only on the predicates (decision points or BCSs) in them.

Cyclomatic Complexity: Example

$v(G) = e - n + 2p$

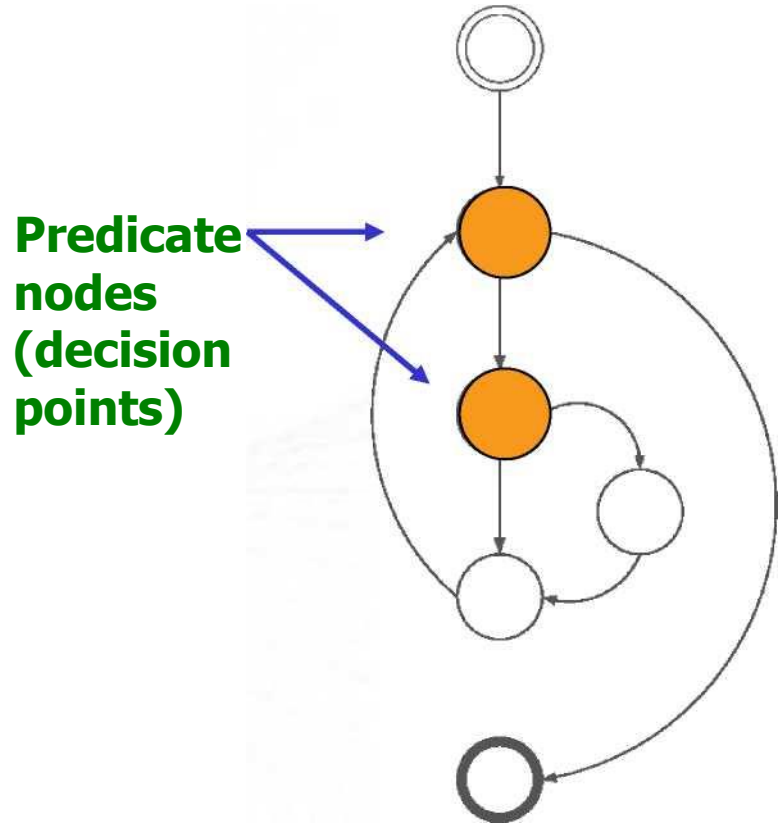
$v(G) = 7 - 6 + 2 \times 1$

$v(G) = 3$

Or

$v(G) = 1 + d$

$v(G) = 1 + 2 = 3$

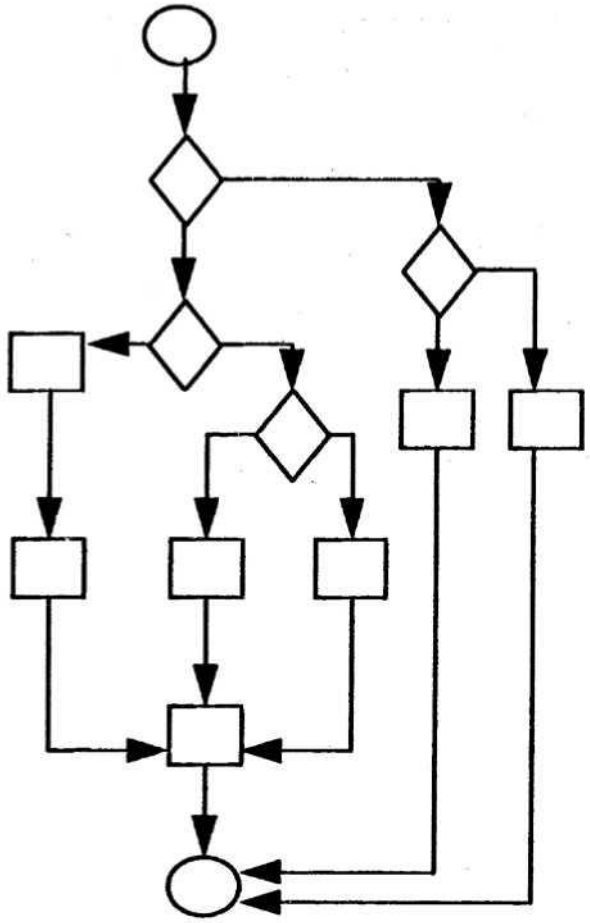


Example: Code Based

```
#include <stdio.h>
main()
{
int a ;
scanf (“%d”, &a);
 if ( a >= 10 )
if ( a   < 20 )   printf   ("10 <   a<   20   %d\n"   ,   a);
else              printf   ("a >=   20   %d\n"   ,   a);
else              printf   ("a <=   10   %d\n"   ,   a);
}
```

$$v(G) = 1+2 = 3$$

Example: Graph Based



v(G) = 16 -13 + 2 = 5

or

v(G) = 4 +1 = 5

Example 1

- Determine cyclomatic complexity for the following Java program:

v = 1+d v =

1+6 = 7

```
01.  import java.util.*;
02.  public class CalendarTest
03.  {
04.  public static void main(String[] args)
05.  {
06.  / / construct d as current date
07.  GregorianCalendar d = new GregorianCalendar();
08.  int today = d.get(Calendar.DAY_OF_MONTH);
09.  int month = d.get(Calendar.MONTH);
10.  / / set d to start date of the month
11.  d.set(Calendar.DAY_OF_MONTH, 1);
12.  int weekday = d.get(Calendar.DAY_OF_WEEK);
13.  / / print heading
14.     System.out.println("Sun  Mon  Tue  Wed  Thu  Fri
Sat");
15.  / / indent first line of calendar
16.  for (int i = Calendar.SUNDAY; i < weekday; i++ )
17.  System.out.print(" ");
18.  do
19.  {
20.  // print day
21.  int day = d.get(Calendar.DAY_OF_MONTH);
22.  if (day < 10) System.out.print(" ");
23.  System.out.print(day);
24.  / / mark current day with *
25.  if (day == today)
26.  System.out.print("* ");
27.  else
28.  System.out.print(" ");
29.  / / start a new line after every Saturday
30.  if (weekday == Calendar.SATURDAY)
31.  System.out.println();
32.  // advance d to the next day
33.  d.add(Calendar.DAY_OF_MONTH, 1);
34.  weekday = d.get(Calendar.DAY_OF_WEEK);
35.  }
36.  while (d.get(Calendar.MONTH) == month);
37.  // the loop exits when d is day 1 of the next month
38.  / / print final end of line if necessary
39.  if (weekday != Calendar.SUNDAY)
40.  System.out.println();
41.  }
42.  }
```

Example 2

- Determine cyclomatic complexity for the following flow diagram:

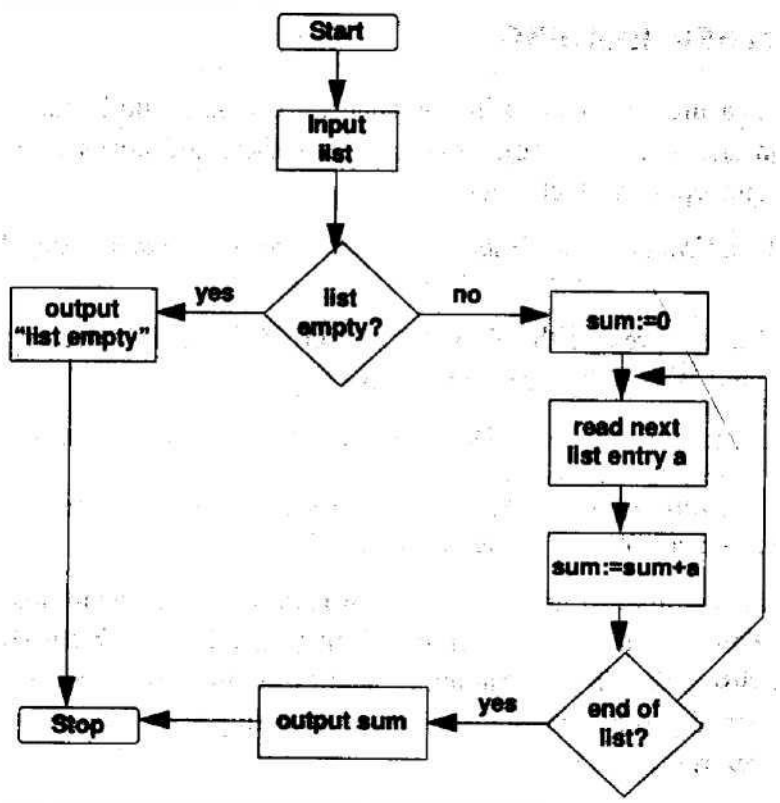
v = 1+d

v = 1+2 = 3

or

v = e - n + 2

v = 11 -10 + 2 = 3



Example 3A

- Two functionally equivalent programs that are coded differently
- Calculate cyclomatic

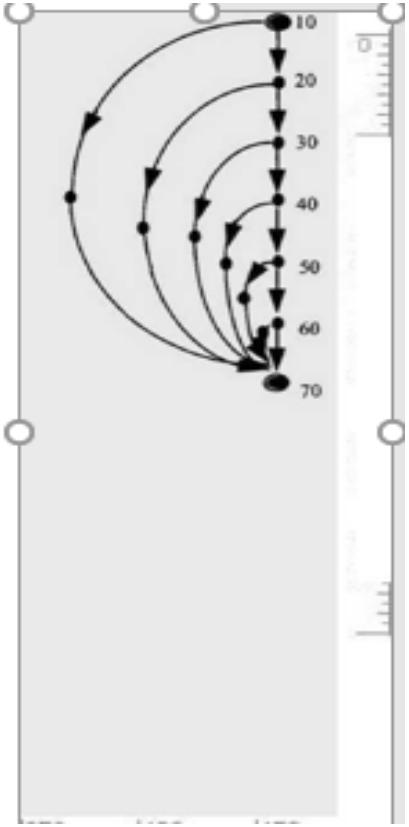
complexity for both

$V_A = 7$

$V_B = 1$

There is always a trade-off between control-flow and data structure. Programs with higher cyclomatic complexity usually have less complex data structure. Apparently program B requires more effort that program A.

```
Program A

10 If score >= 80 Then Print " Grade A": GOTO 70
20 If score >= 70 Then Print " Grade B": GOTO 70
30 If score >= 60 Then Print " Grade C": GOTO 70
40 If score >= 50 Then Print " Grade D"; GOTO 70
50 If score >= 40 Then Print '* Grade E": GOTO 70
60 I f score< 40 Then Print " Grade F": GOTO 70
70 END
```



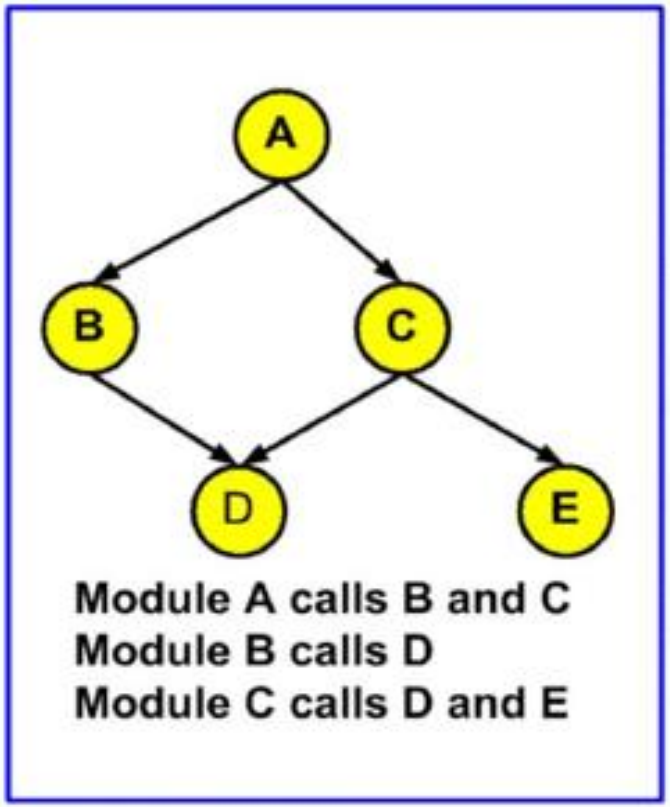

```
Program B

10 DIM A$(11)
20 A$(10) = " Grade A"
30 A$(9) = " Grade A"
40 A$(8) = " Grade A"
50 A$(7) = " Grade B"
60 A$(6) = " Grade C"
70 A$(5) = " Grade D"
80 A$(4) = " Grade E"
90 A$(3) = " Grade F"
100 A$(2) = " Grade F"
120 A$(1) = " Grade F"
130 A$(0) = " Grade F"
140 I = Int (score / 10)
140 Print A$(I)
150 END
```

Cyclomatic Complexity: Critics

- Advantages:
  - Objective measurement of complexity
- Disadvantages:
  - Can only be used at the component level
  - Two programs having the same cyclomatic complexity number may need different programming effort
  - Same requirements can be programmed in various ways with different cyclomatic complexities
  - Requires complete design or code visibility

Software Architecture /1

- A software system can be represented by a graph,
  S = {N, R}
- Each node *n* in the set of nodes (N) corresponds to a subsystem.
- Each edge *r* in the set of relations (R) indicates a relation (e.g., function call, etc.) between two subsystems.



Morphology

- Morphology refers to the overall shape of the software system architecture.
- It is characterized by:
  - Size: number of nodes and edges
  - Depth: longest path from the root to a leaf node
  - Width: number of nodes at any level
  - Edge-to-node ratio: connectivity density measure

Morphology: Example

Size:

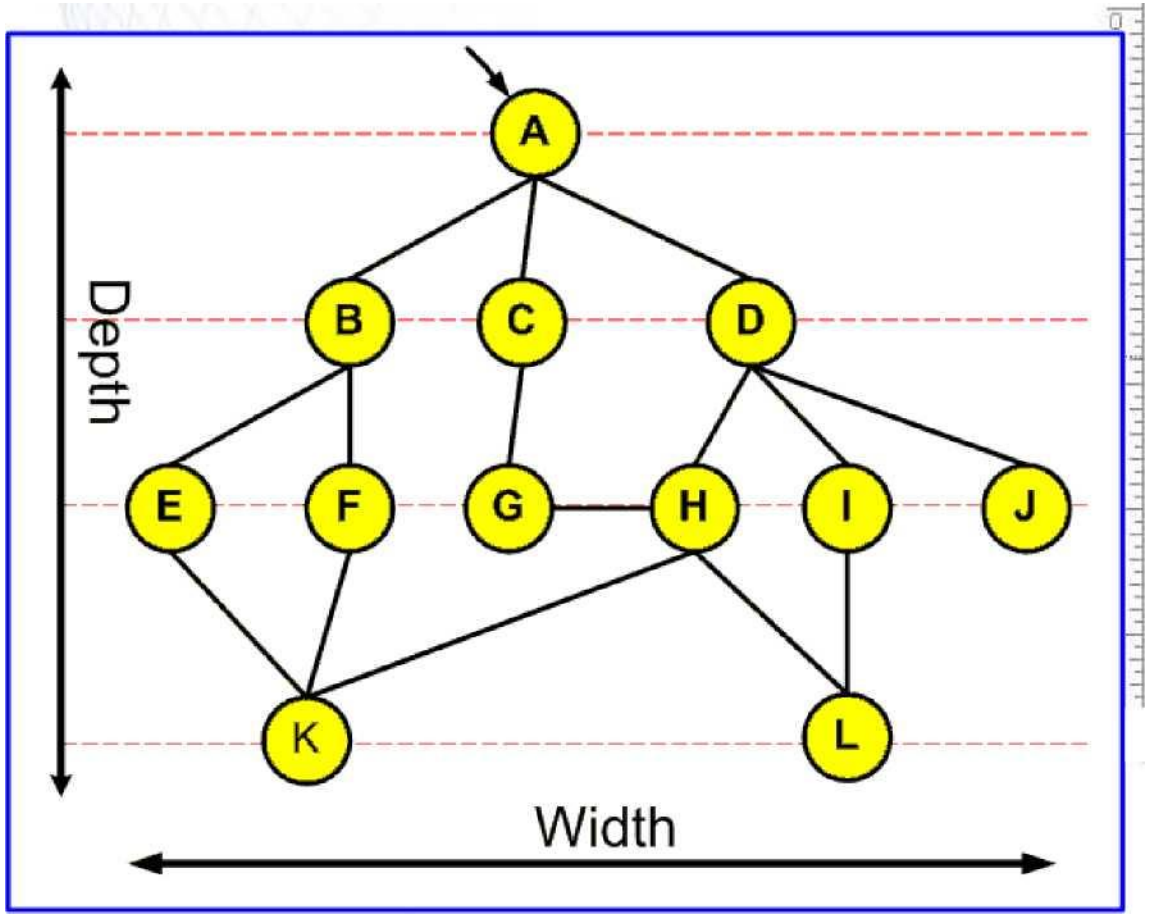
12 nodes

15 edges

Depth: *3*

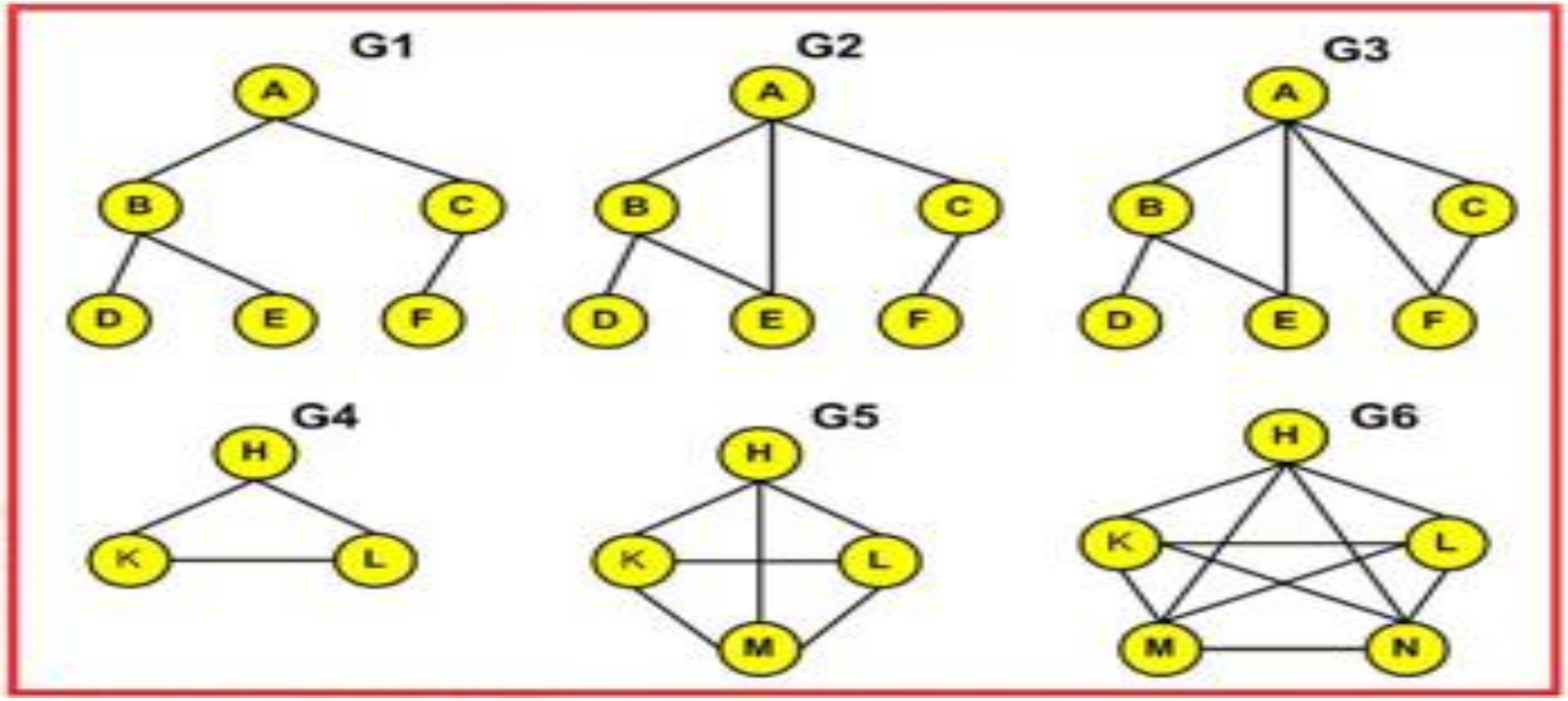
Width: *6*

e/n = 1.25



Tree Impurity /1

- The tree impurity measures *m(G)* how much the graph is different from a tree
- The smaller m(G) donates the better design



Tree Impurity *12*

- Tree impurity can be defined as:

$$m(G) = \frac{\text{number of edges more than spanning tree}}{\text{maximum number of edges more than spanning tree}}$$

$$m(G) = \frac{2(e-n+1)}{(n-1)(n-2)}$$

- **Example:**

m(G1) =0 *m(G2) = 0.1* *m(G3) = 0.2*

m(G4) = 1 *m(G5) = 1* *m(G6) = 1*

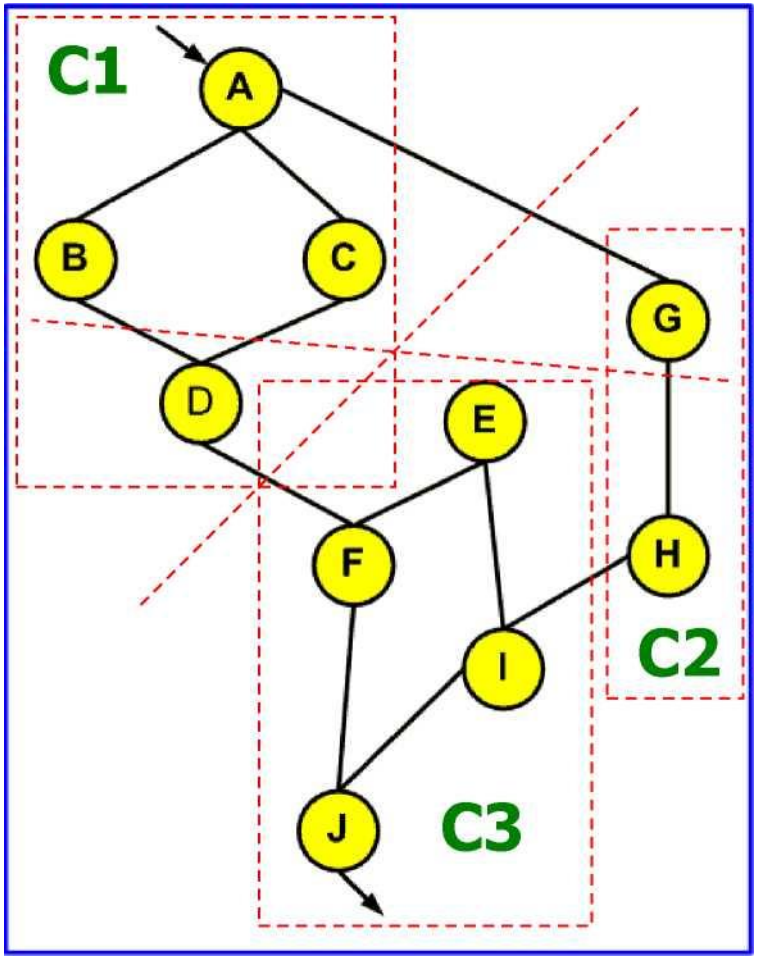
Complexity Measures: Cohesion group 2

Components & Modules

- A **module** or **component** is a bounded sequence of program statements with an aggregate identifier.
- A module usually has two properties:
  - ***Nearly-decomposability property:*** the ratio of data communication within the module is much (at least 10 times) more than the communication with the outside.
  - ***Compilablity property:*** a module should be separately compilable (at least theoretically).

Software Architecture

- A modular or component-based system is a system that all the elements are partitioned into different components.
- Components do not share any element. There are only relationships across the components.

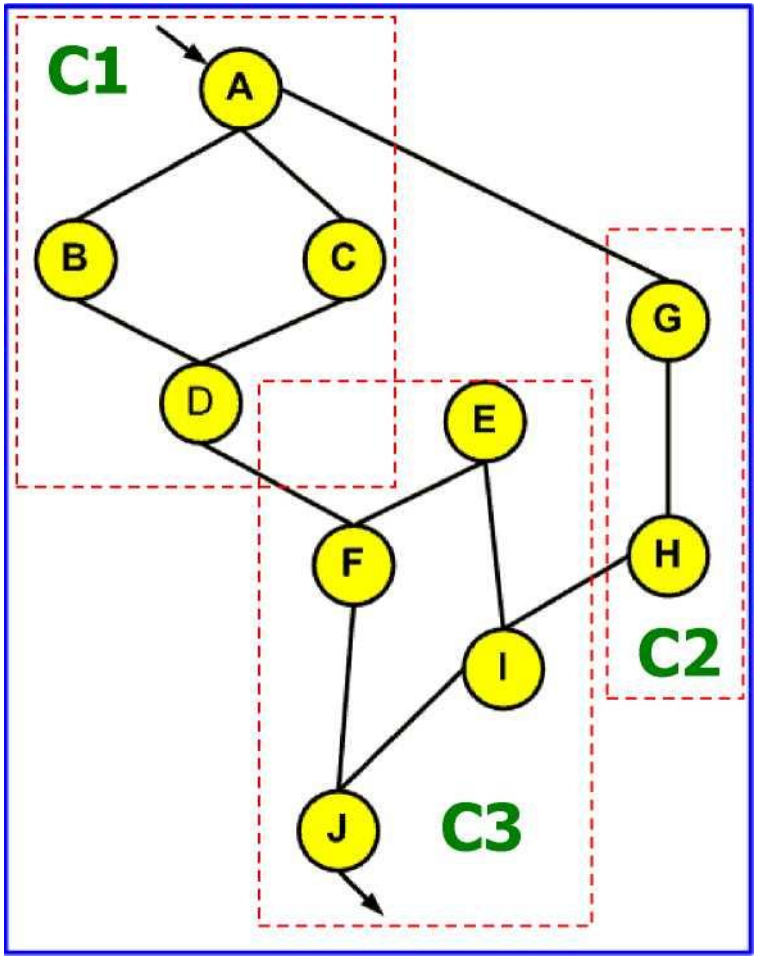


Component Based System (CBS)

- A **component-based system (CBS)** can be defined by a graph, S = {C, R_e}
- Each node *c* in the set of nodes (C) corresponds to a component.
- Each edge *r* in the set of relations (R_e) indicates an external relationship between two components.

CBS: Code Complexity

- Code complexity of a component-based system is the sum of cyclomatic complexity of its components
- Example:
  - *v(C1) = v(C3) = 2*
  - *v(C2) = 1*
  - *v(G) = 2+2+1 = 5*



Concept: Cohesion

- Cohesion describes how strongly related the responsibilities between design elements can be described
- The goal is to achieve “high cohesion”
- High cohesion between classes is when class responsibilities are highly related

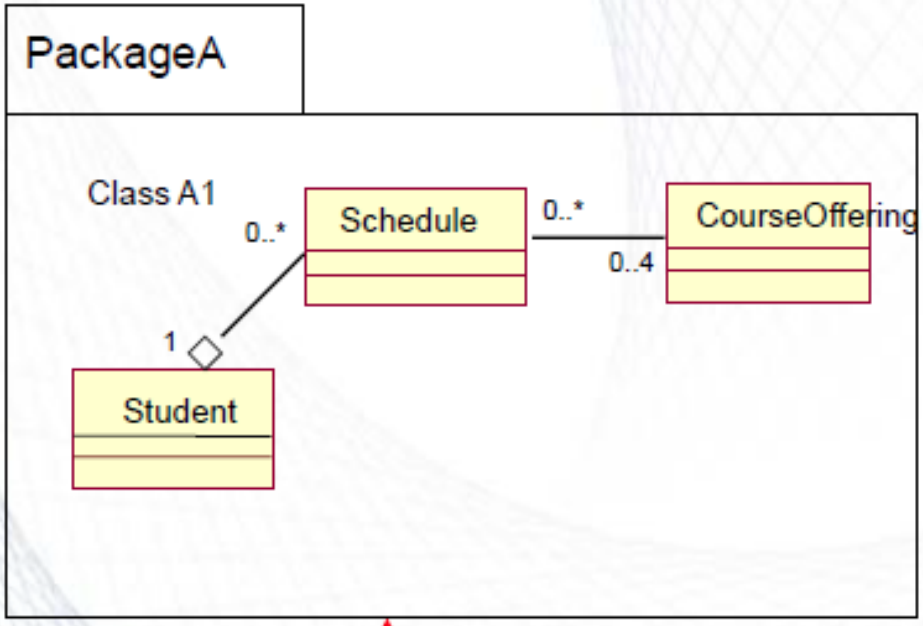
Cohesion /1

- Cohesion of a module is the extent to which its individual components are needed to perform some task.
- Types of cohesion (7 types):
- Functional: The module performs a single function
- Sequential: The module performs a sequence of functions
- Communicative: The module performs multiple function on the same body of data

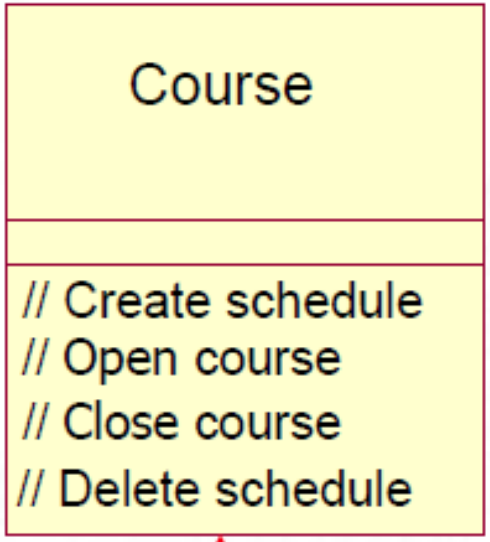
Cohesion /2

- Types of cohesion (cont'd):
  - Procedural: The module performs more than one function related to a certain software procedure
  - Temporal: The module performs more than one function and they must occur within the same time span
  - Logical: The module performs more than one function and they are related logically
  - Coincidental: The module performs more than one function and they are unrelated

Examples: Cohesion



High package cohesion
(functional cohesion)



Low class cohesion
(coincidental and
logical cohesion)

CBS: Cohesion

- **Cohesion** of a module is the extent to which its individual components are needed to perform some task.
- Cohesion for a component is defined in terms of the ratio of internal relationships to the total number of relationships.

$$CH(C_i) = \frac{R_{\text{internal}}}{R_{\text{internal}} + R_{\text{external}}}$$

CBS: System Cohesion

- System cohesion is the mathematical mean of cohesion of all its components.

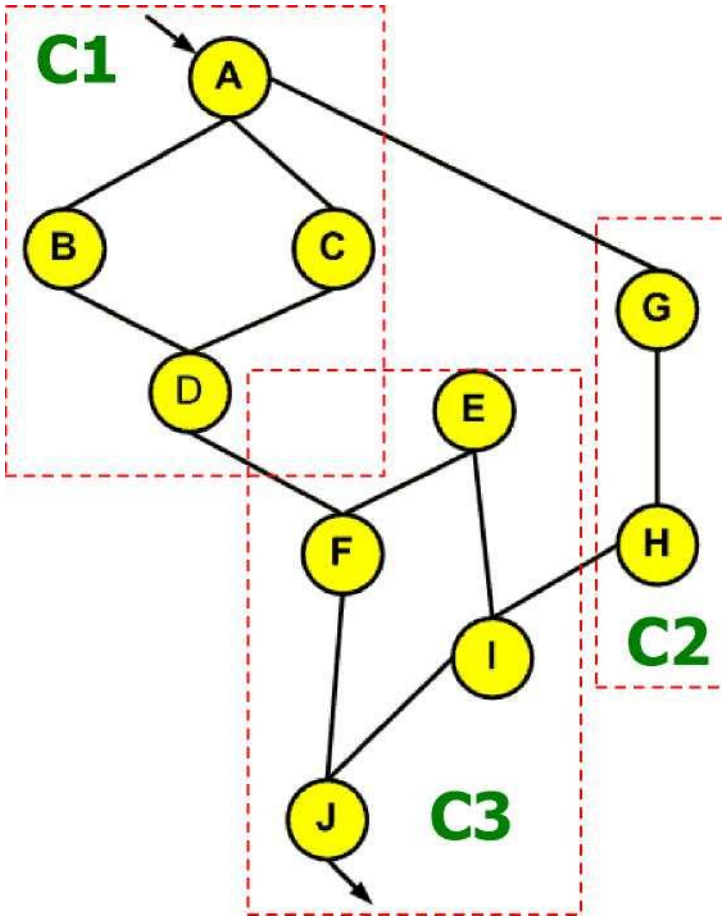
$$CH = \frac{1}{n}\sum_{i=1}^{n} CH(C_i) \qquad 0 \le CH \le \%100$$

- The higher system cohesion is better because it indicates that more job is processed internally.

Example: Cohesion

- Cohesion for a component is defined in terms of the ratio of internal relationships to the total number of relationships.

$$CH(C_i) = \frac{R_{\text{internal}}}{R_{\text{internal}} + R_{\text{external}}}$$



- Example:
  - *CH(C1) =2/3*
  - *CH(C2) = 1/3*
  - *CH(C3) = 2/3*

Example: System Cohesion

- System cohesion is the mathematical mean of cohesion of all its components.

$$CH = \frac{1}{n}\sum_{i=1}^{n} CH(C_i) \qquad 0 \le CH \le \%100$$

- Example:

CH = ((2/3)+(1/3)+(2/3))/3 = 5/9 = %55

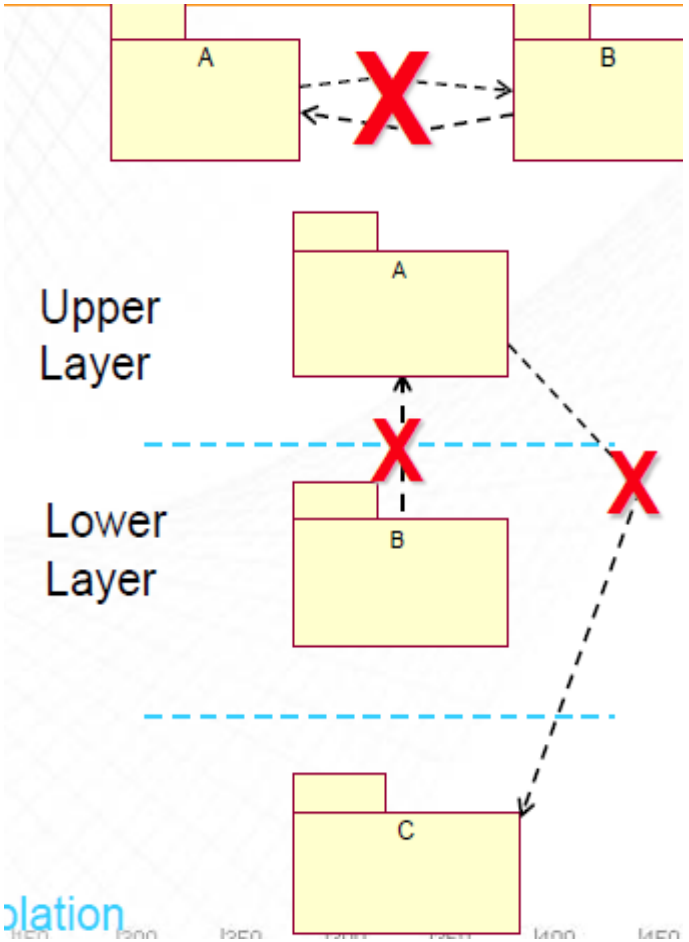
Complexity Measures: Coupling

Concept: Coupling

- Coupling describes how strongly one element relates to another element
- The goal is to achieve “loose coupling”
- Loose coupling between classes is small, direct, visible, and has flexible relations with other classes

Coupling: Package Dependencies

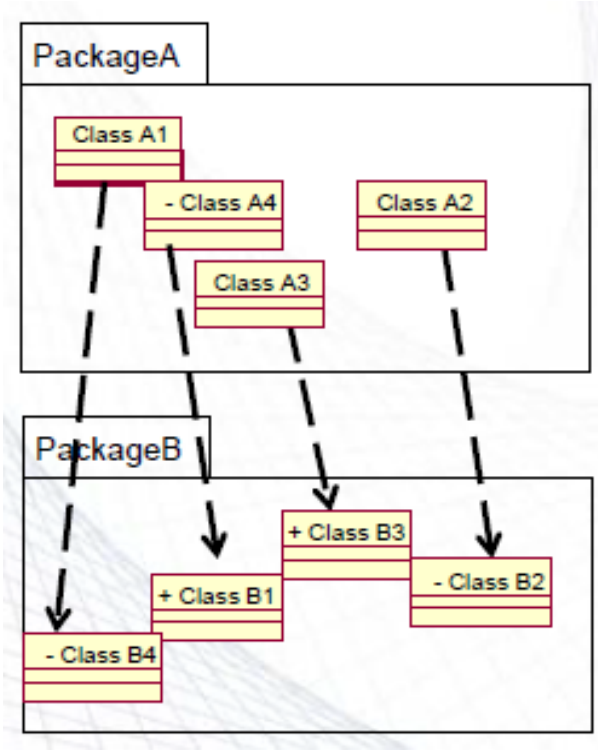
- Packages should not be cross-coupled
- Packages in lower layers should not be dependent upon packages in upper layers
- dependencies should not skip layers (unless specified by the architecture)



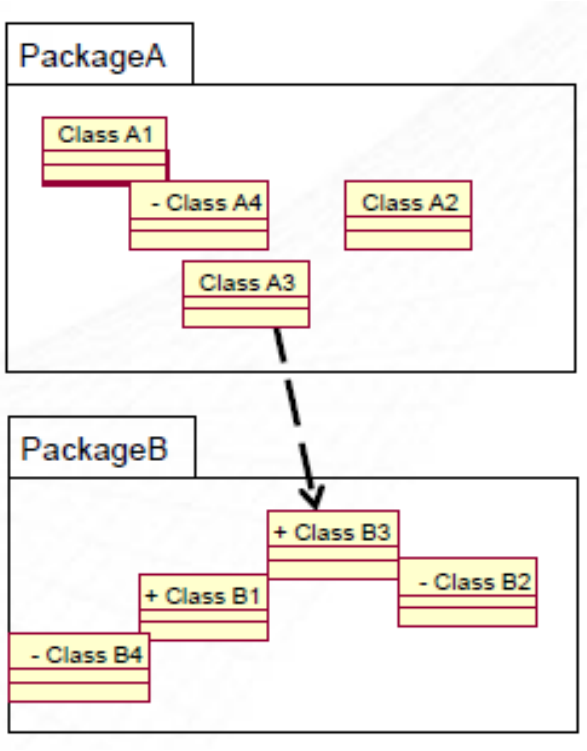
X = coupling violation

Coupling: Class Relationships

- Strive for the loosest coupling possible



Strong Coupling



Loose Coupling

Coupling /1

- Coupling is the ***degree of interdependence*** among modules
  Various types of coupling (5 types):

- - R0: independence: modules have no communication
  - R1: data coupling: modules communicate by parameters
  - R2: stamp coupling: modules accept the same record type
  - R3: control coupling: x passes the parameters to y and the parameter passed is a flag to control the behavior of y.
  - R4: content coupling: x refers to inside of y; branches into or changes data in y.

Coupling *12*

- **No coupling:** R0
- **Loose coupling:** R1 and R2
- **Tight coupling:** R3 and R4

- **There is no standard measurement for coupling!**

CBS: Coupling

- **Coupling** of a component is the ratio of the number of external relations to the total number of relations.

$$CP(C_i) = \frac{\mathrm{R}_{\text{external}}}{R_{\text{internal}} + R_{\text{external}}}$$

CBS: System Coupling

- **System coupling** is the mathematical mean of coupling of all its components.

$$CP = \frac{1}{n}\sum_{i=1}^{n} CP(C_i) \qquad 0 \le CP \le \%100$$

- The lower system coupling is better because it indicates that less effort is needed externally.

Coupling in CBS: Example

- Coupling of a component

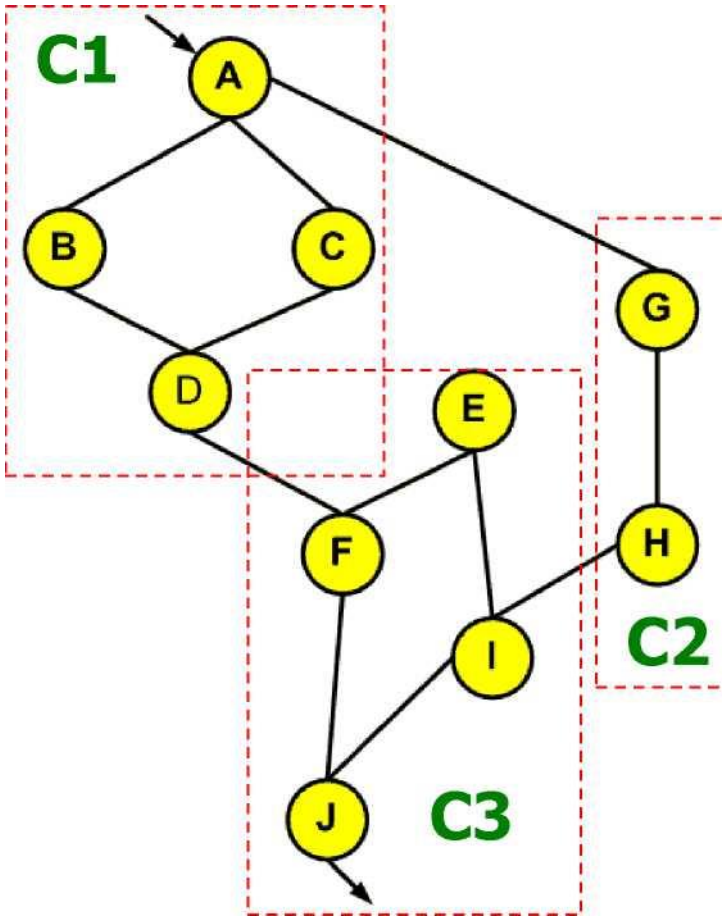
$$CP(C_i) = \frac{R_{external}}{R_{internal} + R_{external}}$$

- Example:

CP(C1) = 1/3

CP(C2)= 2/3

CP(C3)= 1/3



CBS: System Coupling

- **System coupling** is the mathematical mean of coupling of all its components.

$$CP = \frac{1}{n}\sum_{i=1}^{n} CP(C_i) \qquad 0 \le CP \le \%100$$

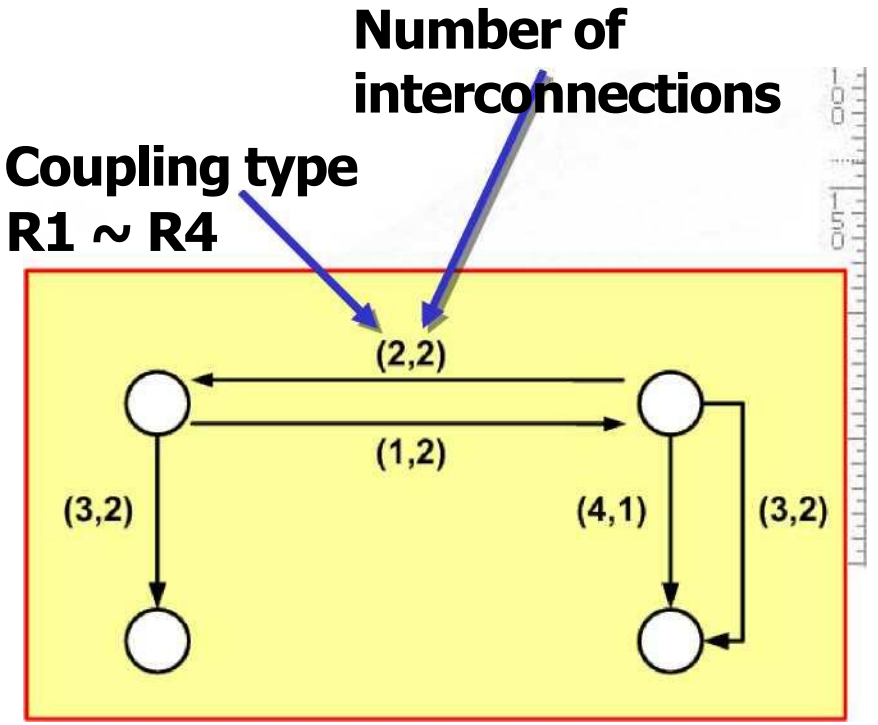
- **Example:**
  - CP = ((1/3)+(2/3)+(1/3))/3 = 4/9 = %45

Coupling: Representation /1

- Graph representation of coupling
- Coupling between modules *x* and *y*:

$$c(x, y) = i + \frac{n}{n+1}$$

- i: degree of worst coupling relation
- n: number of interconnections



Coupling: Representation *12*

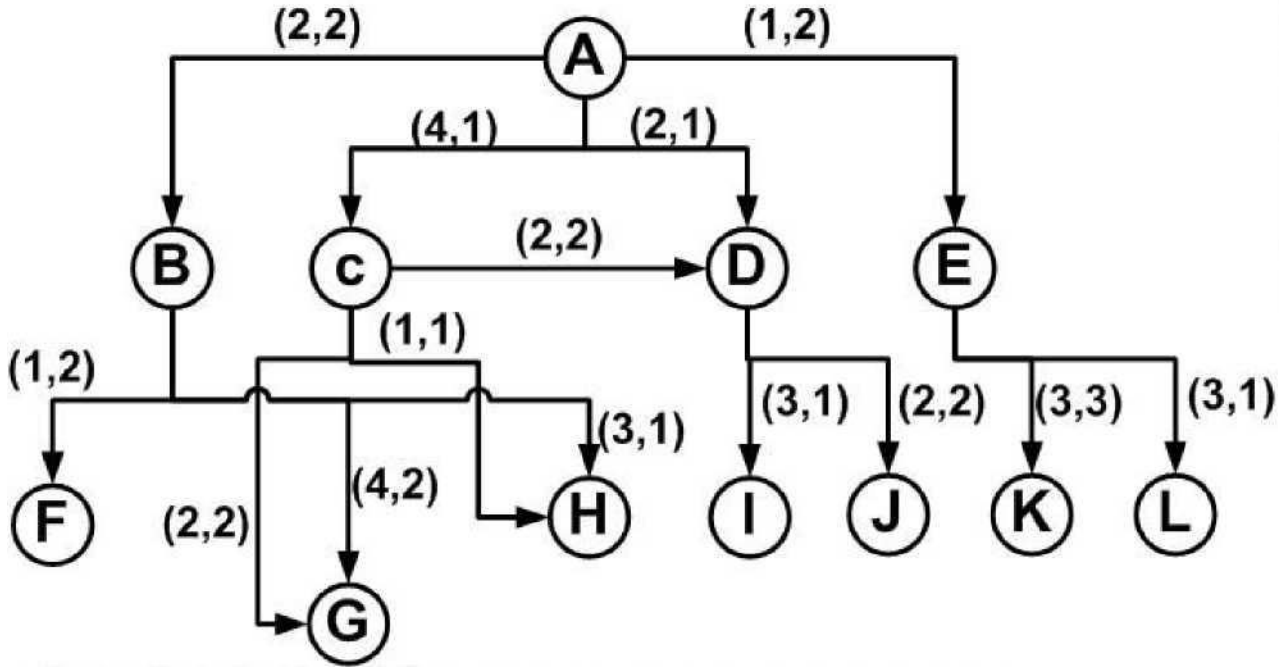
- Global coupling of a system consists of modules is the median value of the set of all couplings c(x,y)

$$C(S) = Med\{c(x,y) \mid \forall x, y \in S\}$$

- Statistical review:
  - Median value: The median value is a measure of central tendency. The median value is the middle value in a set of values. Half of all values are smaller than the median value and half are larger. When the data set contains an odd (uneven) set of numbers, the middle value is the median value. When the data set contains an even set of numbers, the middle two numbers are added and the sum is divided by two. That number is the median value.

Example

- The structural modules of a software system and their interconnections are depicted below. The arrows depict the coupling between modules.



Example (cont'd)

a) Determine the coupling between modules.

b) Determine the global system coupling.

C_{CH}	$= 1 + 1/2 = 1.50$
$C_{AE} = C_{BF}$	$= 1 + 2/3 = 1.66$
C_{AD}	$= 2 + 1/2 = 2.50$
$C_{AB} = C_{CD} = C_{CG} = C_{DJ}$	$= 2 + 2/3 = 2.66$
$C_{BH} = C_{DI} = C_{EL}$	$= 3 + 1/2 = 3.50$
C_{EK}	$= 3 + 3/4 = 3.75$
C_{AC}	$= 4 + 1/2 = 4.50$
C_{BG}	$= 4 + 2/3 = 4.66$

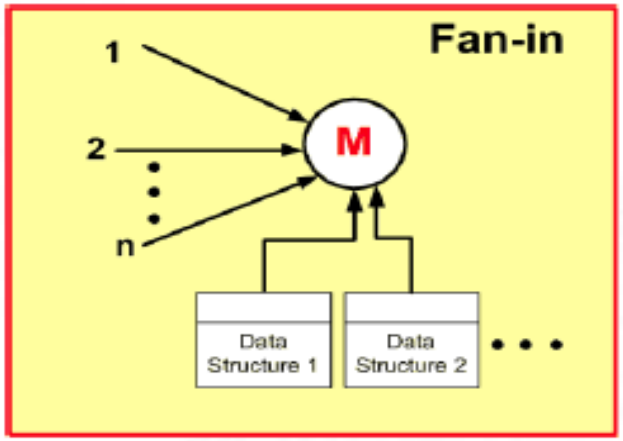
1.50 1.66 1.66 2.50 2.66 2.66 (2.66 2.66) 3.50 3.50 3.50 3.75 4.50 4.66

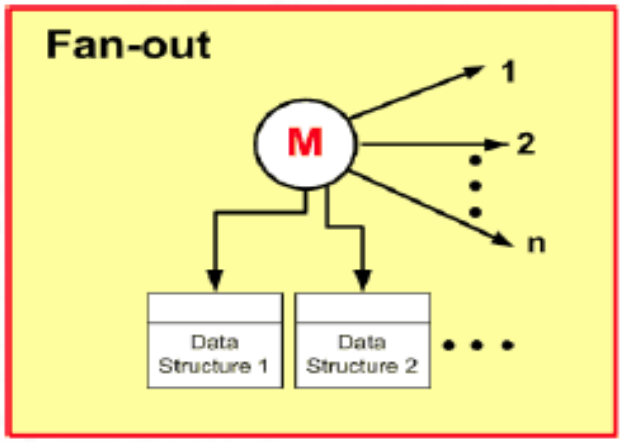
C(S) = (2.66+2.66)/2 = 2.66

Other Structural Metrics

Information Flow Measures /1

- Information flow is measured in terms of fan-in and fan-out of a component.
- Fan-in of a module M is the number of flows terminating at M plus the number of data structures from which info is retrieved by M.
- Fan-out of a module M is the number of flows starting at M plus the number of data structures updated by M.





Information Flow Measures / 2

- Information flow complexity (IF C) [Henry-Kafura, 1981]:

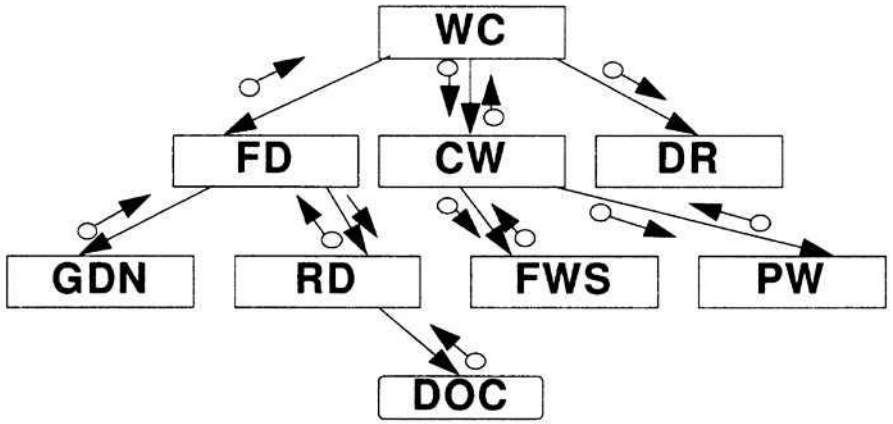
$$IFC(M) = length(M) \times \left(fan\text{-}in(M) \times fan\text{-}out(M)\right)^2$$

- IFC [Shepperd, 1990]:

$$IFC(M) = \left(fan\text{-}in(M) \times fan\text{-}out(M)\right)^2$$

Information Flow Measures /3

- Example



Module	fan-in	fan-out	[(fan-in)(fan-out)]^2	length	'complexity'
WC	2	2	16	30	480
FD	2	2	16	11	176
CW	3	3	27	40	1080
DR	1	0	0	23	0
GDN	0	1	0	14	0
RD	2	1	4	28	112
FWS	1	1	1	46	46
PW	1	1	1	29	29

Information Flow Measures /4

- Revision of the Henry-Kafura IFC measure:
  - Recursive module calls should be treated as normal calls.
  - Any variable shared by two or more modules should be treated as a global data structure.
  - Compiler and library modules should be ignored.
  - Indirect flow's should be ignored.
  - Duplicate flows should be ignored.
  - Module length should be disregarded, as it is a separate attribute.

Data Structure Measurement

- There is always a trade-off between control-flow and data structure.
- Programs with higher cyclomatic complexity usually have less complex data structure.
- A simple example for data-structure measurement:

Integers	$C_1 = n_i \times 1$
Strings	$C_2 = n_s \times 2$
Arrays	$C_3 = n_a \times 2 \times size\ of\ array$
Total	$C = C_1 + C_2 + C_3$

Example 3B

- Calculate date structure complexity for Program A and B

	Program A	Program B
Integers (n_i)	$C1 = 1 \times 1$	$C1 = 1 \times 1$
Strings (n_s)	$C2 = 6 \times 2$	$C2 = 6 \times 2$
Arrays (n_a)	$C3 = 0 \times 2 \times 0$	$C3 = 1 \times 2 \times 11$
Total	13	35

There is always a trade-off between control-flow and data structure. Programs with higher cyclomatic complexity usually have less complex data structure. Apparently program B requires more effort that program A.

```
Program A

10 If score >= 80 Then Print " Grade A": GOTO 70
20 If score >= 70 Then Print " Grade B": GOTO 70
30 If score >= 60 Then Print " Grade C": GOTO 70
40 If score >= 50 Then Print " Grade D"; GOTO 70
50 If score >= 40 Then Print '* Grade E": GOTO 70
60 I f score< 40 Then Print " Grade F": GOTO 70
70 END
```

```
Program B

10 DIM A$( 11)
20  AS(10)  =  "  Grade
A" 30  AS(9) = " Grade
A" 40  A$(8)  = " Grade
A" 50  A$(7)  = " Grade
B" 60  A$(6)  = " Grade
C"  70   AS(5)=" Grade
D" 80  A$(4)  = " Grade
90 AS(3)=" Grade F"
100 A$(2) = " Grade F"
120 AS(1) = " Grade F"
130 A$(0) * " Grade F'
140 I = Int (score / 10)
140 Print A$(I)
150 END
```